

Question 1 [4+2+2 pts]: Let $g(x) = \begin{cases} (2x+1)\cos x & \text{if } x \leq 0 \\ \frac{1-\sqrt{x+1}}{x} & \text{if } x > 0. \end{cases}$

- (a) Show that g is discontinuous at $x = 0$.
- (b) Use part (a) to determine if g is differentiable at $x = 0$.
- (c) Find $g'(x)$, if $x < 0$.

Question 2 [2 pts]: Show that the equation $x^3 + 2x^2 + 5x = -7$ has at least one real solution.

Question 3 [2+2 pts]: Consider the function $g(x) = (\sin(\pi x) + \sqrt{x})^3$.

- (a) Find $g'(x)$.
- (b) Find an equation for the tangent line to the curve $y = g(x)$ at $x = 1$.

Question 4 [2+2 pts]: Let $f(x) = \frac{\sqrt{x^2+1}}{x-3}$. Find all vertical and horizontal asymptotes, if any.

Question 5 [2 pts]: Let f be a function satisfying

$$4x - 9 \leq f(x) + x \leq x^2 - 4x + 7, \text{ for all } x \text{ in } (-\infty, \infty).$$

Find $\lim_{x \rightarrow 4} f(x)$ (if it exists).

Question 6 [5 pts]: The graph of a function $y = f(x)$ is given below.

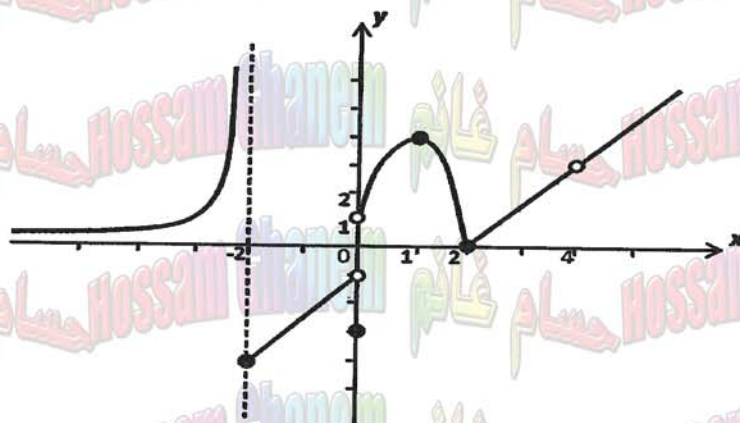


Figure 1:

Use the graph to answer the following questions. Justify your answers.

- (a) Find $\lim_{x \rightarrow -\infty} f(x)$
- (b) What type of discontinuity does f have at $x = -2$?
- (c) What type of discontinuity does f have at $x = 0$?
- (d) What type of discontinuity does f have at $x = 4$?
- (e) Is f differentiable at $x = 2$?